Séminaire de géométrie algébrique de Rennes [1]

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Résumé : A homogeneous polynomial in $N+1$ variables is said to have vanishing hessian if the determinant of its Hessian matrix is zero. Thus these are exactly the homogeneous polynomial solutions of the classical Monge-Ampère differential equation and a solution is called trivial if it depends, modulo a linear change of coordinates, on $r \leqslant N$ variables. Hesse published two papers claiming that there exist only trivial homogeneous polynomial solutions to the classical Monge-Ampere equation ; or equivalently that the associated hypersurfaces in $\mathbf{P}^N$ are cones. The question is quite subtle because, as it was firstly pointed out by P. Gordan and M. Noether in 1876, Hesse's claim is true for $N \leqslant 3$ and in general false for $N \geqslant 4$. The cases $N = 1, 2$ (or $d = 2$) are easily handled but beginning from $N = 3$ the problem is related to non trivial characterizations of cones among hypersurfaces having zero gaussian curvature at every regular point. The aim of the talk is to present from scratch the circle of ideas originated by geometrical and algebraic interpretations of Hesse's claim in different areas of mathematics. Then we shall illustrate : the proof of Hesse's claim for $N \leqslant 3$ according to Gordan and Noether ; some series of examples of hypersurfaces with vanishing hessian, not cones, for every $N \geqslant 4$; the classification in low dimension and/or low degree based on a geometrical interpretation of Gordan-Noether analysis.

1. Les jeudis matin, de 10 h 30 à 11 h 30, salle 004, IRMAR (bâtiment 22), Université de Rennes 1, Campus de Beaulieu