Hilbert's Tenth Problem for function fields over valued fields

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Abstract

A diophantine equation is a polynomial equation with integer coefficients in an arbitrary number of variables. Hilbert's Tenth Problem was the following question: find an algorithm which, given a diophantine equation, tells whether or not it has a solution over the integers. It was shown in 1970 by Y. Matiyasevich, building on earlier work by M. Davis, H. Putnam and J. Robinson, that this question has a negative answer: such an algorithm does not exist. In other words: diophantine equations are undecidable.

Many authors have generalized this undecidability from the integers to other rings and fields. Two important open cases are $\mathbb{Q}$ and $\mathbb{C}(x)$. In this talk, I will give an overview of what is known and then show that equations over $\mathbb{C}((t))(x)$ are undecidable. This is a special case of a more general theorem which works for function fields over a valued field in equal characteristic zero having certain properties. The proof generalizes methods by Kim and Roush (1992) who showed the analogous result for $\mathbb{C}(x_1, x_2)$. Elliptic curves and quadratic forms play an important role in the proof.