Hodge structure on the fundamental group and its application to p-adic integration

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Abstract

We study the unipotent completion $\Pi_{un}^{DR}(x_0, x_1, X_K)$ of the de Rham fundamental groupoid [De] of a smooth algebraic variety over a local non-archimedean field K of characteristic 0. We show that the vector space $\Pi_{un}^{DR}(x_0, x_1, X_K)$ carries a certain additional structure. That is a $\mathbb{Q}_p^{ur}$-space $\Pi_{un}(x_0, x_1, X_K)$ equipped with a σ-semi-linear operator ϕ, a linear operator N satisfying the relation $N\phi = p\phi N$ and a weight filtration $W_\bullet$ together with a canonical isomorphism $\Pi_{un}^{DR}(x_0, x_1, X_K) \otimes_K \overline{K} \simeq \Pi_{un}(x_0, x_1, X_K) \otimes_{\mathbb{Q}_p^{ur}} \overline{K}$. We prove that an analog of the Monodromy Conjecture holds for $\Pi_{un}(x_0, x_1, X_K)$.

As an application, we show that the vector space $\Pi_{un}^{DR}(x_0, x_1, X_K)$ possesses a distinguished element. In the other words, given a vector bundle E on X_K together with a unipotent integrable connection, we have a canonical isomorphism $E_{x_0} \simeq E_{x_1}$ between the fibres. The latter construction is a generalisation of Colmez's p-adic integration ($rk\, E = 2$) and Coleman's p-adic iterated integrals (X_K is a curve with good reduction).

In the case, when X_K is proper and has a smooth model over the ring of integers $R \subset K$, a similar construction has been independently found by Besser