Design des automates algébriques pour les implémentations hardwares et softwares en cryptographie symétrique.

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Common work with
François Arnault, Cédric Lauradoux, Marine Minier, Benjamin Pousse, Gaël Thomas, ...

Main publications:

- Arnault F., Berger T. P., Minier M., Pousse B.: Revisiting LFSRs for Cryptographic Applications, IEEE Transactions on Information Theory , 57(12), p.8095-8113 (2011)
- Arnault F., Berger T. P., Lauradoux C., Minier M., Pousse B. A New Approach for FCSRs, In Michael J. Jacobson Jr., Vincent Rijmen, Reihaneh Safavi-Naini editors, Selected Areas in Cryptography - SAC 2009,LNCS 5867: 433-448, Springer 2009.
- Arnault F., Berger T. P., Pousse B.: A matrix approach for FCSR automata, Cryptography and Communications, v.3 (2): p.109-139 (2011)

Autonomous LFSM

An Autonomous Linear Finite State Machine (LFSM) of length n, with ℓ outputs consists of:

- A set of n cells, $m = (m_0, \ldots, m_{n-1}) \in \mathbb{F}_2^n$, called the set of *states* of the automaton.
- A linear transition function from $\mathbb{F}_2^n$ to $\mathbb{F}_2^n$.
- A linear extraction function from $\mathbb{F}_2^n$ to $\mathbb{F}_2^\ell$.

T: $n \times n$ matrix of the transition function,
C: $n \times \ell$ matrix of the extraction function,

- Initialization: state $m(0) \in \mathbb{F}_2^n$ at time $t = 0$
- From the state $m(t)$ at time t, output: $v(t) = Cm(t)$
- Compute a new state $m(t+1) = Tm(t)$

An example: LFSR in Fibonnaci mode

$$T_1 = \begin{pmatrix} & 1 & & & & & & \\ & & 1 & & & (0) & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & (0) & & & & 1 & \\ & & & & & & & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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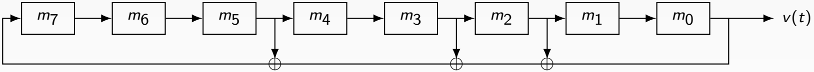

Another example: LFSR in Galois mode

$$T_2 = \begin{pmatrix} 0 & 1 & & & & & & \\ 0 & & 1 & & & (0) & & \\ 1 & & & 1 & & & & \\ 0 & & & & 1 & & & \\ 1 & & & & & 1 & & \\ 1 & & (0) & & & & 1 & \\ 0 & & & & & & & 1 \\ 1 & & & & & & & \end{pmatrix}$$

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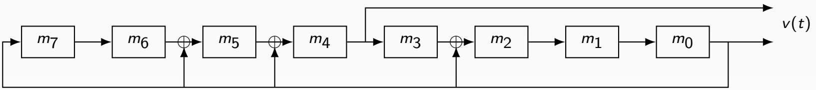

A third example: LFSR in Ring mode

$$T_3 = \begin{pmatrix} & 1 & & & & & & \\ & & 1 & & & (0) & & \\ & & & 1 & & & & \\ & & & & 1 & & & 1 \\ & & & & & 1 & & \\ & & (0) & & & & 1 & \\ & & & & & & & 1 \\ 1 & & 1 & & & & & \end{pmatrix}$$

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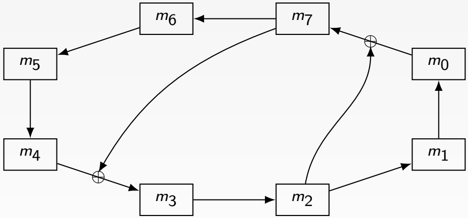


Output of LFSMs

$M_i(X) = \sum_{t=0}^{\infty} m_i(t)X^t$
$M = (M_0(X), ..., M_{n-1}(X))$

Theorem

If the initial state of a LFSM is $m = (m_0, ..., m_{n-1})$ *then*

$$^tM = \frac{Adj(I - XT)}{q(X)}\ ^tm$$

with $q(X) = \det(I - XT)$.

If $\det(T) \neq 0$ and $q(X)$ primitive, then

$M_i(X) = p_i(X)/q(X)$ m-sequences of period $2^n - 1$.

Examples continued

for $i = 0$, 2 or 3, $\quad q_i(X) = \det(I - XT_i) = X^8 + X^6 + X^5 + X^3 + 1$

$Adj(I - XT_3) =$

$$\begin{pmatrix}
x^6+x^3+1 & x^7+x^4+x & x^2 & x^3 & x^4 & x^5 & x^6 & x^7+x^4 \\
x^7+x^4 & x^6+x^3+1 & x & x^2 & x^3 & x^4 & x^5 & x^6+x^3 \\
x^6+x^3 & x^7+x^4 & 1 & x & x^2 & x^3 & x^4 & x^5+x^2 \\
x^5+x^2 & x^6+x^3 & x^7+x^5+x^4+x^2 & 1 & x & x^2 & x^3 & x^4+x \\
x^4 & x^5 & x^6+x^4 & x^7+x^5 & x^5+x^3+1 & x^6+x^4+1 & x^7+x^5+x^2 & x^3 \\
x^3 & x^4 & x^5+x^3 & x^6+x^4 & x^7+x^5 & x^5+x^3+1 & x^6+x^4+x & x^2 \\
x^2 & x^3 & x^4+x^2 & x^5+x^3 & x^6+x^4 & x^7+x^5 & x^5+x^3+1 & x \\
x & x^2 & x^3+x & x^4+x^2 & x^5+x^3 & x^6+x^4 & x^7+x^5 & 1
\end{pmatrix}$$

LFSRs: for what purpose?

Non cryptographic context

- Simulation: hight speed for random numbers generation (Monte Carlo method...)
- Initialization tests of arithmetic circuits in computers
- Implementation of counters...

Cryptographic context

- basic building block for the design of automata in symmetric cryptography
- Good statistical properties
- Proved period...

Depending on the target, 2 types of outputs

- One bit output
- Block of bits output

Efficient implementations

Software applications

- Use the natural bloc structure (8, 16, 32, 64 bits) of the processor and assembly instructions
- Minimize the cycles: pipe-line optimizations, etc...

Hardware applications

- Power consumption
- Area of the circuit, number of gates
- Minimize path, fan-out...

A new concept: diffusion delay

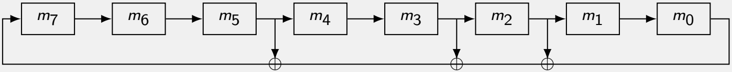
The diffusion delay is the smallest number d, such that there exist two cells m_i and m_j with the following property: the successive values $m_j(0)$, ..., $m_j(d-1)$ are independent of the value $m_i(0)$.

Definition

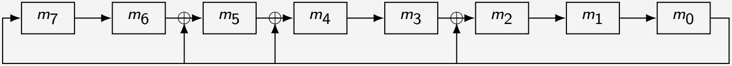
Diffusion delay = Diameter of the graph of connection of the cells

One bit output: hardware implementations

- Fibonnaci



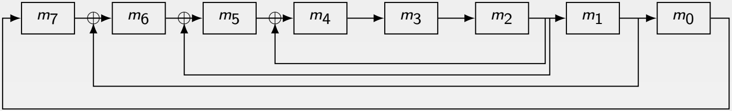
- Galois



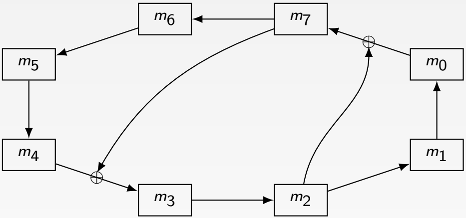
	Critical path	Fan-out	Cost	Diffusion delay
Galois	1	$\approx n/2$	$\approx n/2$	$n-1$
Fibonacci	$\approx n/2$	2	$\approx n/2$	$n-1$

One bit output: hardware implementations

- G. Mrugalski, J. Rajski, and J. Tyszer, 2004



- Ring



	Critical path	Fan-out	Cost	Diffusion delay
Mrugalski & all	2	3	$\approx n/2$	$\approx n/2$
Optimal Ring	1	2	$\approx n/2$	$\approx n/4$

Words oriented software implementations

- Twisted Generalized Feedback Shifts Registers
  Matsumoto & Kurita 1992, Matsumoto & Nishurima 1998

$$A = \begin{pmatrix} 0 & I_w & & & & \\ & 0 & I_w & & (0) & \\ & & 0 & I_w & & \\ & (0) & & \ddots & \ddots & \\ & & & & 0 & I_w \\ I_w & 0 & \cdots & L & 0 & 0 \end{pmatrix}$$

I_w: $w \times w$ identity matrix, L: a $w \times w$ binary matrix.

Words oriented software implementations

- Multiple-Recursive Matrix Method H. Niederreiter 1995, see also Marsaglia 2003 (Xorshift PRNG)

$$A = \begin{pmatrix} 0 & I_w & & & & \\ & 0 & I_w & & (0) & \\ & & 0 & I_w & & \\ & (0) & & \ddots & \ddots & \\ & & & & 0 & I_w \\ A_r & A_{r-1} & A_{r-2} & \dots & A_2 & A_1 \end{pmatrix}$$

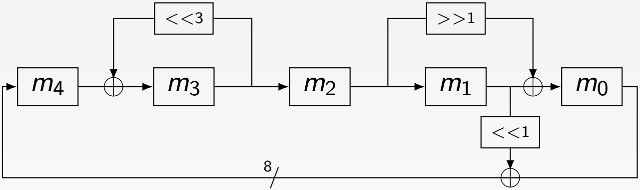
I_w: $w \times w$ identity matrix, A_i: software efficient transformations (right or left shifts, word rotations).

$$q(X) = \det(I - XA) = \det\left(I + \sum_{j=1}^{r} X^j A_j\right)$$

Word-oriented ring LFSRs

- F. Arnault, T.P. B., M. Minier, P. Pousse, 2011

A small example: $A = \begin{pmatrix} & I_8 & R^1 & & \\ & & I_8 & & \\ & & & I_8 & \\ & & & L^3 & I_8 \\ I_8 & L^1 & & & \end{pmatrix}$



Optimal, both in hardware and software.

Problem How to construct good word-ring LFSRs ?

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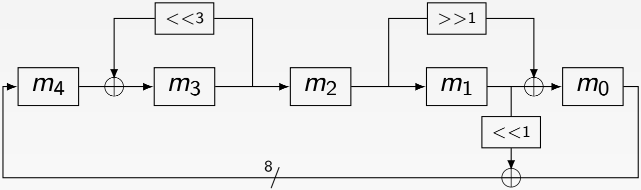

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Problem How to construct good word-ring LFSRs ?

l-adic topology

$\mathcal{A}$:unitary commutative ring. $\mathcal{I} = < \pi >$ such that

- π is not a 0 divisor.
- $\bigcap_{n \in \mathbb{N}} \mathcal{I}^n = \{0\}$

Ultrametric distance:

$$d(x,y) = \begin{cases} 2^{-k} & \textit{if} \quad x \neq y \\ 0 & \textit{if} \quad x = y \end{cases} \quad \text{with } k = \max\{n \in \mathbb{N},\ x - y \in \mathcal{I}^n\}$$

Associated l-adic (π-adic) topology.

Topologic completion of $\mathcal{A}$: $\mathcal{A}_{\mathcal{I}}$ (or $\mathcal{A}_{\pi}$).

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mod_π **function**

Suppose that there exist 2 functions

$$\mathrm{mod}_\pi : \mathcal{A} \longrightarrow \mathcal{S} \subseteq \mathcal{A} \qquad \mathrm{div}_\pi : \mathcal{A} \longrightarrow \mathcal{A}$$

such that

$$a = \pi\, \mathrm{div}_\pi(a) + \mathrm{mod}_\pi(a) \text{ for all } a$$

Set $\mathcal{S} = \mathrm{mod}_\pi(\mathcal{A})$.

More requirement:

- $\mathcal{S}$ is a set of representatives of $\mathcal{A}/(\pi)$
- $0, 1 \in \mathcal{S}$.

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Convergence

For $a \in \mathcal{A}$, set $\mathrm{seq}_\pi(a) = (s_n)_{n\in\mathbb{N}}$ with

$$s_n = \mathrm{mod}_\pi(\mathrm{div}_\pi{}^n(a)).$$

Theorem

Set $a \in \mathcal{A}$ and $s = \mathrm{seq}_\pi(a)$. The series $\sum_{n\in\mathbb{N}} s_n\pi^n \in \mathcal{A}_\pi$ is convergent in $\mathcal{A}_\pi$, moreover $a = \sum_{n\in\mathbb{N}} s_n\pi^n$.

“Integers”

Integers

$$\mathcal{F} = \{a \in \mathcal{A}_\pi \mid \exists n \in \mathbb{N}^*, {\mathrm{div}_\pi}^n(a) = 0\}.$$

Signed integers

$$\mathcal{Z} = \{a - b \mid a, b \in \mathcal{F}\}.$$

arithmetic

If $\mathcal{S}$ is finite, elements of $\mathcal{Z}$ are representable on computers

If $s + t$ and st are in $\mathcal{Z}$ and known, it is possible to provide an effective arithmetic on $\mathcal{Z}$.

Periodic elements

Set $\mathcal{P}$ the set of periodic elements of $\mathcal{A}_\pi$, i.e. such that $\mathrm{seq}_\pi(a)$ is ultimately periodic.

Lemma

$$\mathcal{P} = \{p = a' p(T) + a''\}$$

with a', $a'' \in \mathcal{Z}$ *and* $p(T) = \sum_{i=0}^{\infty} p^{iT}$ $(= 1/(1 - p^T))$.

Proposition

$$\mathcal{Z} \subseteq \mathcal{P}.$$

Rational elements

Set $\mathcal{Q} = \{u/v \mid u,\ v \in \mathcal{Z},\ v \text{ invertible in } \mathcal{Z}_\pi\}$.

Lemma

$a \in \mathcal{Z}$ is invertible in $\mathcal{Z}_\pi$

$$\Leftrightarrow$$

$s_0 = \mathrm{mod}_\pi(a)$ is invertible in $\mathcal{Z}$.

Proposition

$$\mathcal{Q} = \{u/(1+\pi v) \mid u,\ v \in \mathcal{Z}\}$$

Proposition

$$\mathcal{P} \subseteq \mathcal{Q}.$$

AFSM automata

Definition

An algebraic automata on $\mathcal{Z}$ of size $n \in \mathbb{N}^*$ is constituted of

- a set of states $(m, c) \in \mathcal{S}^n \times \mathcal{A}_\pi^n$
- a transition function given by a $n \times n$ matrix T with coefficients in $\mathcal{A}_\pi$.

If the automaton is in the state $m(t), c(t)$ at times t, then

$$\begin{cases} z(t+1) & = & Tm(t) + c(t) \\ m(t+1) & = & \text{mod}_\pi(z(t+1)) \\ c(t+1) & = & \text{div}_\pi(z(t+1)) \end{cases}$$

AFSM automata

$M(t) = (M_0(t), ..., M_{n-1}(t))$ is the n-tuple of π-adic integers observed in the cells m_0, m_{n-1} from time t.

Proposition

$$M(t+1) = TM(t) + c(t).$$

Theorem

If the automaton is in state (m, c) at time t_0, then

$$M(t_0) = \frac{\mathrm{adj}(I - \pi T)}{\det(I - \pi T)} \cdot (m(t_0) + \pi c(t_0))$$

Problem: an algebraic automaton is not necessary finite.

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Example: $\mathcal{A} = \mathbb{F}_2[x]$

- $\mathcal{A} = \mathbb{F}_2[x]$, $\pi = x$, T with coefficients in $\mathbb{F}_2$

  $\Rightarrow$ classical binary LFSRs (or LFSMs).

- $\mathcal{A} = \mathbb{F}_2[x]$, $\pi = x^d$, T with coefficients $t_{i,j}(x)$, $deg(t_{i,j}) < d$

  $\Rightarrow$ d-parallelized binary LFSRs .

- $\mathcal{A} = \mathbb{F}_2[x]$, $\pi = x$ and T with rational coefficients

  $\Rightarrow$ Global definition of binary LFSRs .

An example of global description

$$T = \begin{pmatrix} 0 & P_1(X)/Q_1(X) & 0 \\ 0 & 0 & P_2(X)/Q_2(X) \\ P_3(X)/Q_3(X) & 0 & 0 \end{pmatrix}$$

$$\det(I - XT) = \frac{Q_1(X)Q_2(X)Q_3(X) + X^3 P_1(X)P_2(X)P_3(X)}{Q_1(X)Q_2(X)Q_3(X)}$$

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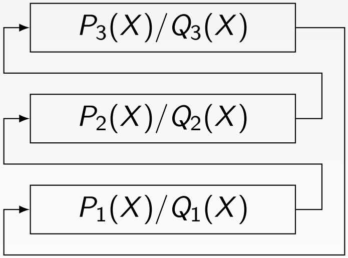

Example: $\mathcal{A} = \mathbb{Z}$

- $\pi = 2$, T binary

  $\Rightarrow$ 2-adic integers, classical FCSRs (Feedback with Carry Shift Registers).

- $\pi = 2$, T with coefficients in $\mathbb{Z}$: a more general framework
  can be always realized with binary FCSRs

All the software or hardware oriented design of LFSRs can be directly applied to FCSRs!

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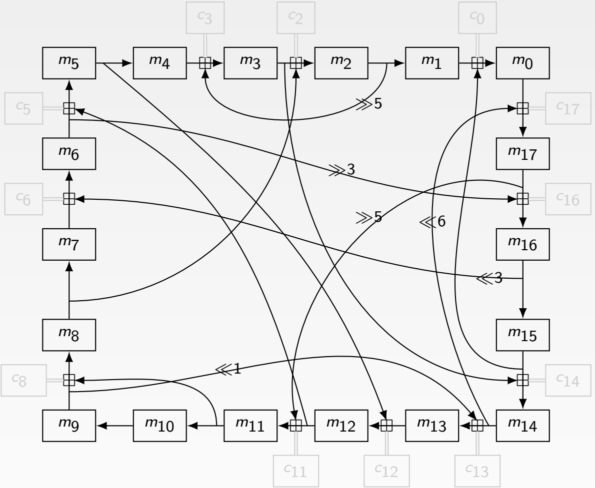
FCSR automaton for GLUON 64



18 blocs of 8 bits

FCSR of 144 bits

+ 73 bits of carries

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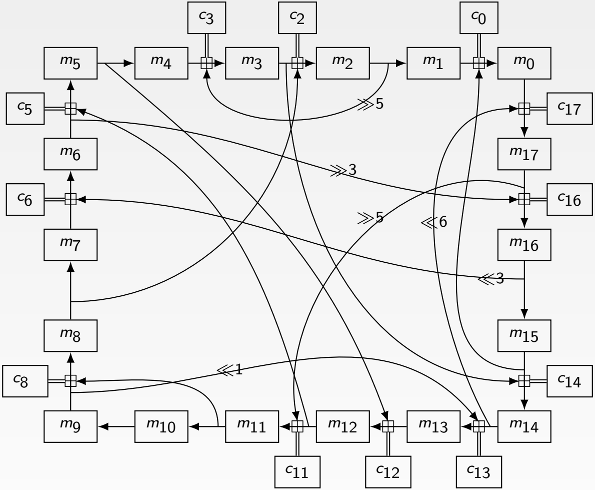

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Corresponding matrix

$$T = \begin{pmatrix}
0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\
0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & SR^5 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & SL^3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & SR^5 \\
0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & SL^1 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & SR^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \\
I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & SL^6 & 0 & 0 & 0
\end{pmatrix}$$

Example: $\mathcal{A} = \mathbb{Z}[x]/p(x)$, $p(x)$ unitary

- $p(x) = x^d - \sum_{i=0}^{d-1} \epsilon_i x^i$, $\epsilon_i \in \{0, 1\}$, $\pi = 2$.

  $\Rightarrow$ V-FCSRs introduced by Allailou, Marjane and Mokrane for Galois and Fibonnaci mode.
  Can be generalized to any mode.
  In fact, the practical implementation of V-FCSRs is nothing else than non-optimal binary FCSRs. [1]

- $p(x) = x^d - n$, $\pi = x$.

  $\Rightarrow$ $\mathcal{A} = \mathbb{Z}[\sqrt[d]{n}]$ and $\pi = \sqrt[d]{n}$. Generalization introduced by Klapper et Goresky.
  Decimation of sequences leads to classical FCSRs.

[1] Berger T.P., Minier M., Cryptanalysis of Pseudo-random Generators Based on Vectorial FCSRs, Indocrypt 2012, Kolkata.

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A new example?

$\mathcal{A} = \mathbb{Z}[x]/p(x)$, $p(x) = \pi(X)^d - N$ unitary

- In this case, AFSM are finite automata
- It seems that they cannot be reduced to classical FCSRs.
- Reconstruction algorithms for such sequences?